Efficient descriptions of unary languages by context-free grammars and pushdown automata

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It is well-known that context-free languages defined over a one-letter alphabet are regular. This implies that unary context-free grammars and unary pushdown automata are equivalent to finite automata.

In this talk, we present these equivalences from a descriptional complexity point of view. In particular, we show optimal upper bounds for the number of states of nondeterministic and deterministic finite automata equivalent to unary context-free grammars in Chomsky normal form. These bounds are functions of the number of variables of the given grammars. We also give upper bounds for the number of states of finite automata simulating unary pushdown automata.

There are some consequences in space complexity: we are able to prove a $\log \log n$ lower bound for the workspace used by one-way auxiliary pushdown automata in order to accept nonregular unary languages. The notion of space we consider is the so-called *weak space* concept.

We consider also transformations of finite automata into context-free grammars: we show that any unary deterministic automaton with n states can be simulated by a context-free grammar with $O(n^{\frac{1}{3}})$ variables and that any unary deterministic automaton with n states can be simulated by a context-free grammar with $O(n^{\frac{2}{3}})$. Similar results do not hold for larger input alphabets.

References:

G. Pighizzini, J. Shallit, M.-w. Wang. Unary Context-Free Grammars and Pushdown Automata, Descriptional Complexity and Auxiliary Space Lower Bounds. *Journal of Computer and System Sciences* 65: 393-414, 2002.

M. Domaratzki, G. Pighizzini, and J. Shallit. Simulating finite automata with context-free grammars. *Information Processing Letters* 84: 339-344, 2002.